

# ANNEX 7

## BACKGROUND INFORMATION AND APPLICATION OF THE SUBSURFACE TEMPERATURE WORKFLOW

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# 1. Introduction, lessons learned

## 1.1. Joint standards and settings

The aim of the presented workflow is to produce the following output datasets:

- Mean annual surface temperature map;
- Temperature gradient maps;
- Subsurface temperature maps;

These outputs will provide additional information for estimating the heat transfer rate of borehole heat exchangers for both, standard web maps at predefined settings and the interactive web data query. The updated workflows intend to provide a joint framework in the project GeoPLASMA-CE for approaches of different complexity. It may consider the use of measured temperature profiles but also provides methods for pure synthetic models if measured subsurface temperatures are missing.

The subsurface temperature is necessary to calculate the power and energy potential for borehole heat exchangers (BHE). For preparing the output, the following joint criteria and default settings were defined:

- Reference BHE length: 100 meters.
- Minimum BHE length: 50 meters.
- Maximum BHE length: depending on maximal depth of the models in a specific pilot area.
- Aimed maximum accuracy of the estimated subsurface temperature:  $\pm 0.5$  degC.
- Aimed maximum accuracy of the estimated gradients:  $\pm 0.05$  degC/100m.

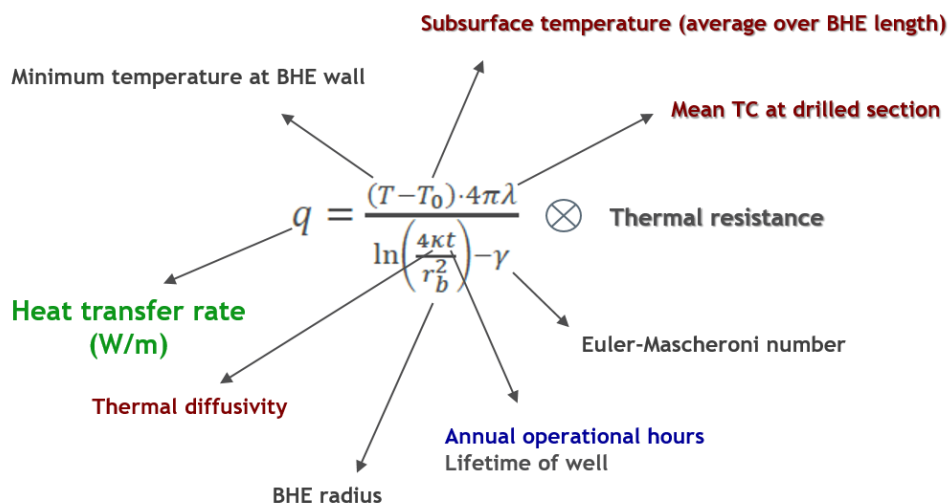
## 1.2. Fundamental considerations

The basic information to project a geothermal installation is to know the heating and cooling demand of the planned buildings. After estimation of the SPF (seasonal performance factor) and COP (coefficient of performance) of the heat pump the energy and power demand for the geothermal system can be determined. In case of using a BHE (borehole heat exchanger) system, the planner wants to know the amount and depth of the needed BHEs to guarantee a proper operation over the planned life time. For dimensioning of the BHE system the planner needs to know the average subsurface temperature and the average thermal conductivity along the planned length (depth) of the BHE.

It is common practice to estimate the **subsurface temperature** by building a linear model and by assuming the geothermal gradient in combination with the actual surface temperature. Therefore the **surface temperature and the mean gradient** are essential parameters to calculate the average subsurface temperature along the planned BHE. The questions are: “How to know the geothermal gradient?” and “Is the actual surface temperature a proper starting temperature, or if climate change effects or urban heat island effect has to be considered?”

The initial version of the GeoPLASMA-CE workflows just considered the parameter “surface temperature” to indicate resources of closed loop systems. During the parameter studies and sensitivity studies, we learned that a good estimation of the subsurface temperature is vital for evaluating the capacity of a borehole heat exchanger, expressed by the specific heat transfer rate (W/m). According to the line-source theory, the average subsurface temperature for a defined depth interval shows the same sensitivity as the thermal conductivity of the ambient rocks, see Figure 1.

## Heat transfer rate of BHE units



**Figure 1: Estimation of the heat transfer rate based on the line source theory. The initial mean subsurface temperature and the mean thermal conductivity are the two sensitive parameters depending on the location of the BHE**

This simple consideration is valid for locations, where the heat transport in the earth is dominated by heat conduction or, in other words, without significant groundwater flow. At locations with significant groundwater flow the “mean effective thermal conductivity” has to be considered. The effective thermal conductivity is a virtual conductivity which includes also effects of groundwater flow (convection and advection). The “mean effective thermal conductivity” can be determined at existing BHE by performing a thermal response test.

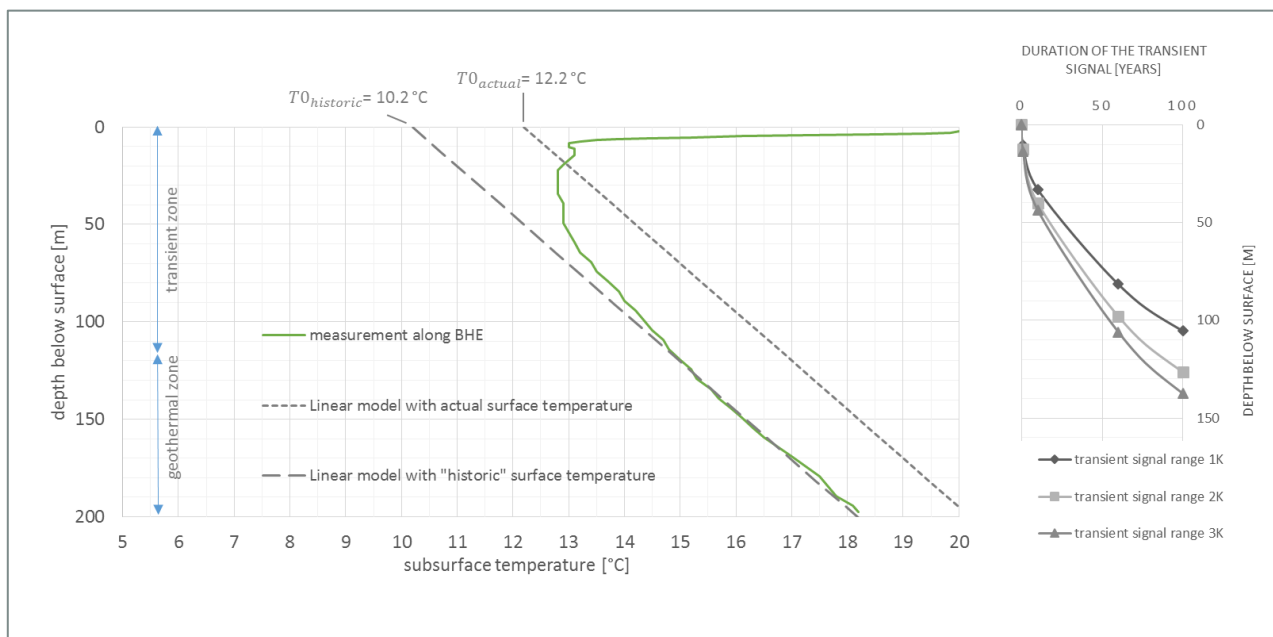
Besides the location-dependent parameters, also operation-dependent parameters (life time, operation hours, constant or interrupted operation) and geometric parameters (BHE diameter, arrangement of BHE pipes, BHE materials, distance to other installations) influences the BHE dimensioning a little.

**LESSON 1: The two most sensitive location-dependent parameters for designing a BHE system are “mean subsurface temperature” and “mean effective thermal conductivity” along the BHE**

As already mentioned, it is common practice to estimate the **subsurface temperature** by assuming the geothermal gradient in combination with the actual surface temperature. This can lead to an overestimation of the mean subsurface temperature, as effects of climate change and urban development of the last century are neglected. Figure 2 demonstrates the situation at one temperature-depth profile, measured at a BHE near the central railway station in Vienna: The gradient zone begins at about 115 m below surface. Prolonging this linear gradient to the surface

gives the historic surface temperature of 10.2 °C. Above the geothermal zone it can be assumed, that the linear temperature is disturbed by a transient signal of the last decades. Comparing the right diagram of Figure 2, the transient signal needs a duration of about 100 years with an amplitude of 1 Kelvin, or a duration of about 60 years with an amplitude of 2-3 K to reach the depth of 115 m. The historic surface temperature of the basic linear model is determined by extrapolation of the gradient to the surface and gives 10.2 °C, in this case. This coincides well with the actual surface temperature of 12.2 °C, when the climate change effect of the last 60 years of about 2 K is considered (compare with Figure 17).

**LESSION 2: The surface starting point of the linear model of the gradient zone is generally not the actual mean surface temperature**

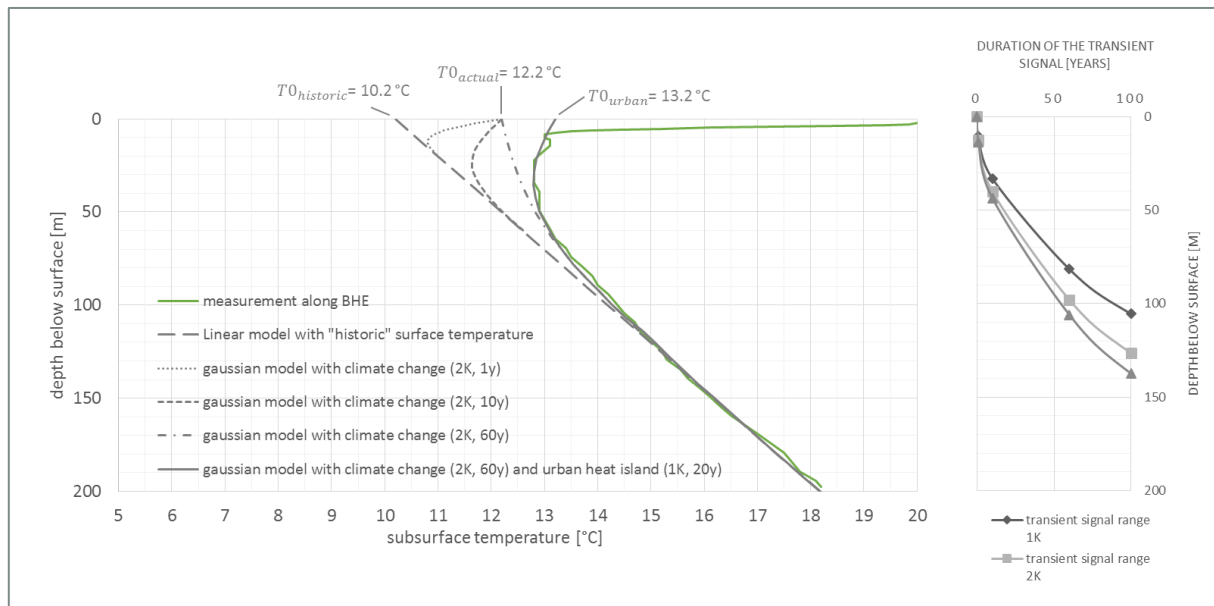


**Figure 2: Example at a measured temperature profile in the city of Vienna #1. LEFT: The gradient zone begins in about 115 m depth. The surface starting point of the linear model of the gradient zone is in general not the actual mean surface temperature, which is at this location is 12.2 °C (annual average). RIGHT: The depth range of a transient signal, with an amplitude of 1-3 Kelvin, in dependence of its duration.**

Figure 3 demonstrates the application of the Gaussian model (calculation of a transient surface temperature change, (theoretical background see chapter 3.2) to the linear model at the example in Vienna: The measured temperature profile can be explained best by adding a transient signal of 2 Kelvin for the last 60 years (corresponding to climate change effect, see Figure 17) overlapped by an additional transient signal of 1 Kelvin for the last 20 years (assumed to be urban heat island effect).

With the help of the electronic ANNEX 6b of the guidelines the Gaussian model can help to estimate the effect and depth range of a transient signal.

**LESSION 3: The effect to the subsurface of a transient signal at the surface can be calculated by applying the Gaussian model to the linear model as a good approximation.**



**Figure 3: Example at a measured temperature profile in the city of Vienna #2. The measured temperature profile (solid green) can be explained best by the gaussian model with a transient signal of 2 K of the last 60 years added with a second transient signal of 1 Kelvin for the last 20 years (solid grey). Other gaussian models consider a 2 Kelvin surface rise for the last 1 year, 10 and 60 years in comparison. As longer the transient signal lasts, the deeper the signal range.**

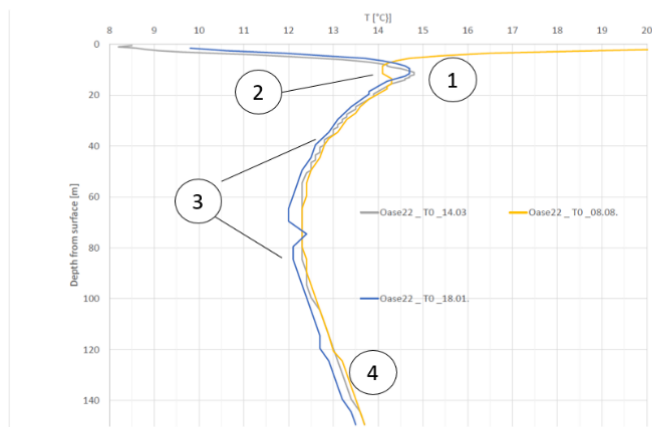
**LESSION 4:** The best way to determine the basic linear model (geothermal gradient and historic surface temperature) is from measured temperature profiles. Therefore it is necessary, that the measured profile reaching depths greater than the transient zone. In most areas, it can be assumed in a first estimation that the temperature measurements needs to reach depths of 100 to 200 m to get proper gradient information.

### 1.3. Urban vs. non-urban

As already mentioned, transient temperature changes of the last century at the surface effects the subsurface temperature regime of the first 100 meters. This transient effect is in general higher in urban areas, than in non-urban areas. Therefore it is more important to consider the transient disturbances in urban areas. Figure 4 gives two example of measured temperature profiles. The left profile was taken in urban area (Vienna), on a location with the highest mean annual groundwater temperature (groundwater body in about 10-25 m depth). The transient signal reaching a depth of about 70 m. The right diagram shows a temperature profile in a non-urban area, where no transient disturbance of the subsurface temperature can be observed.

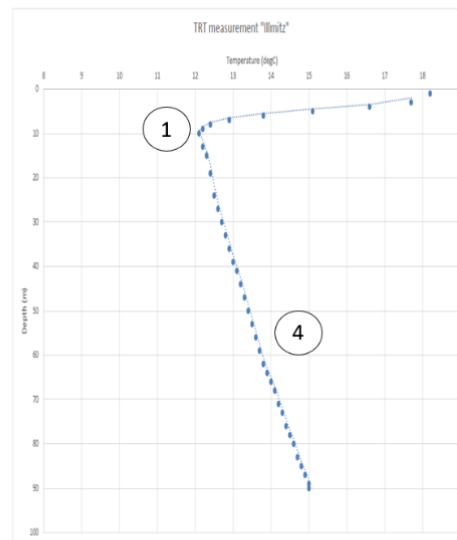
Due to the fact, that transient temperature changes in non-urban areas cannot be precluded, the workflow to produce the output maps is more dependent on the availability of information about transient temperature changes rather than urban or non-urban location.

#### Urban temperature profiles (Vienna)



Temperature profile at the location “Oase 22p” in the city of Vienna, derived from repeated temperature profile measurements at a TRT site.

#### Non-urban temperature profiles (Burgenland)



Temperature profile below the small village “Illmitz” in Burgenland, derived from a single temperature profile measurement at a TRT site.

**Figure 4: Comparison of specific subsurface temperature profiles in urban (left) and non-urban (right) regions. 1: Seasonal zone; 2: groundwater influenced by urbanization or industry; 3: Transient zone; 4: Geothermal zone.**

## 1.4. Planned outputs in the pilot areas

For all pilot areas, the “Mean annual surface temperature” is planned to show on the web-based information system.

Furthermore, the partners realized the individual outputs in the pilot areas, given in Table 2.

In contrast to the initial planning no partner of the GeoPLASMA-CE project decided to display maps of the effective thermal gradient on the web-based information system.

**Table 1: Overview of the realized outputs in the GeoPLASMA-CE pilot areas.**

Pilot area	Effective thermal gradient	Interval subsurface temperature	Depth intervals
Vogtland / W-Bohemia	No	No	None
Walbrzych / Broumov	No	Yes	0 to 100, 0 to 200
Krakow	No	Yes	0 to 50, 0 to 100, 0 to 150, 0 to 200
Vienna	No	Yes	0 to 50, 0 to 100, 0 to 150, 0 to 200
Bratislava/ Hainburg	No	Yes	0 to 100, 0 to 150
Ljubljana	No	Yes	0 to 50, 0 to 100, 0 to 150, 0 to 200



## 1.5. Requirements for creating subsurface temperature maps

To generate subsurface temperature maps or gradient maps the following information is essential:

- surface temperature map
- gradient information of the geothermal zone, best given by measurement, secondary given as a result of modelling (numerically or analytically) or tertiary given by simple estimation.
- additional temperature measurements in the transient zone, if temperature changes at the surface or other thermal disturbances (e.g. due to industry or urbanization) are
- Optionally historic air temperature information, to estimate the “climate change” effect.

## 1.6. Data

## 2. Demonstration of the harmonised workflow at PA Vienna

This chapter will demonstrate the performance of the elaborated standard workflow to create subsurface temperature and/or gradient map at the example of the pilot area Vienna (PA Vienna). The surface temperature maps is an input to generate the subsurface temperature maps. The workflow consists of five main steps, given in Figure 5.

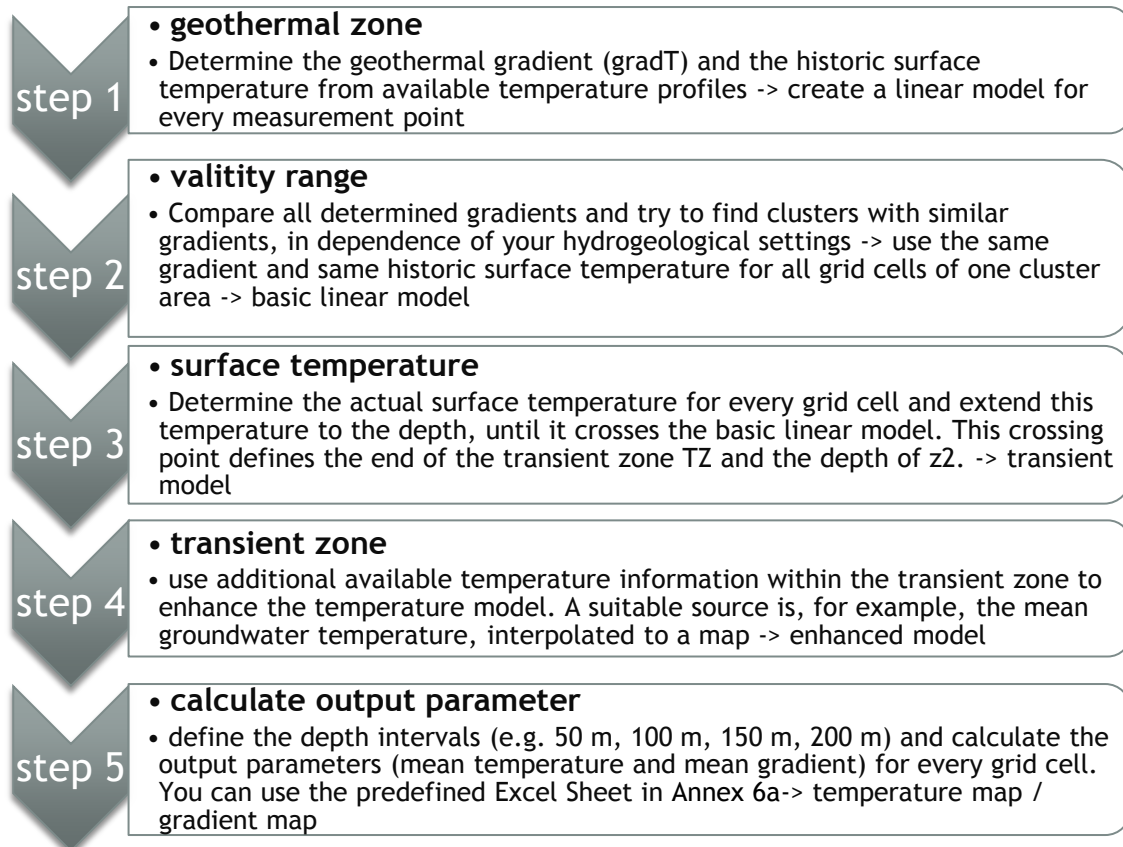


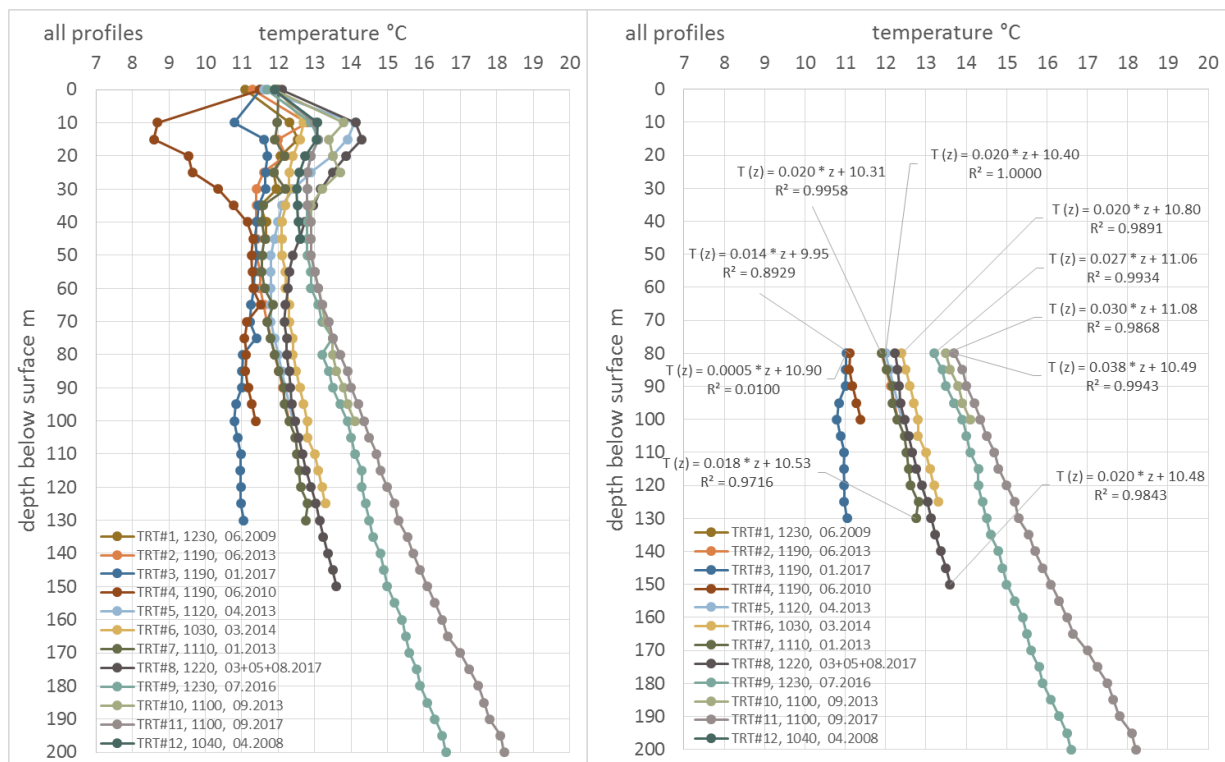
Figure 5: Adapted workflow for calculating the average interval subsurface temperature in the pilot area Vienna.

### 2.1. step 1: geothermal zone

In Vienna 14 temperature profiles are available, measured between 2008 and 2017. 2 of them are rejected from analysis due to obviously unreasonable temperatures. The remaining 12 temperature profiles are plotted in Figure 6 (left diagram), whereby the temperature profiles are already resampled in 5 m depth steps and the temperature at the surface is replaced by the actual mean surface temperature at the location of the log. The transient zone can already be estimated visually, reaching depths up to 80 m.

In a next step, the gradient of every log is calculated with data beginning at 80 m depth. Afterwards, the **linear model** is extrapolated to the surface ( $z = 0$  m) to get the “historic” surface temperature ( $T0\_historic$ ). In GeoPLASMA-CE, we applied a quality criteria of  $R^2 > 0.95$  to be suitable. Now, a linear model ( $T0\_historic + \text{gradient}$ ) can be assigned to every temperature log and we are ready for the next workflow step.





**Figure 6: Plot of all available temperature profiles in Vienna (LEFT) and gradient analysis of the profiles (RIGHT)**

## 2.2. step 2: validity range

There are different approaches for determining the validity area of the gradients:

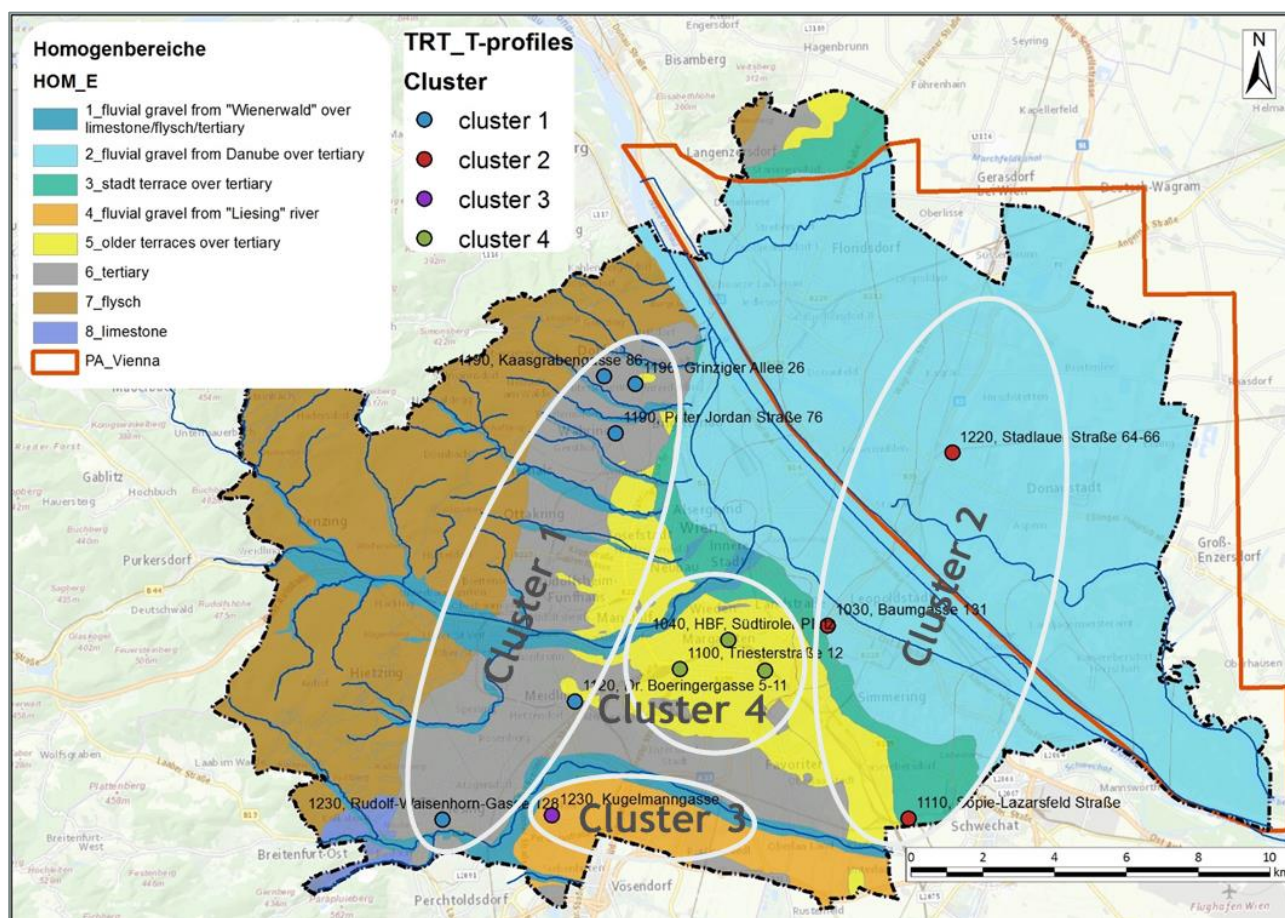
- *Case 1 - no direct measurements of subsurface profiles are available:* without qualified measurements the gradient has to be estimated. This can be done by applying simple analytical models as the Gaussian model - you can use the Excel tool in the Annex 6b. The challenge is to find the correct historic surface temperature, which is very sensitive to the virtual temperature profile. Also the thermal underground parameters have to be estimated. Numerical heat transport models can handle detailed underground parameters but have the same challenge as the analytic Gaussian approach.
- *Case 2 - only one or a low number of temperature profile measurements are available:* We recommend performing spatial cluster analyses and define zones with similar gradient and historic surface temperature, by considering hydrogeological information.
- *Case 3 - high number of temperature profile measurements (ratio between number of grid cells and observation point is less than 1,000):* A gradient map can be created and directly be upscaled from single observation points by raster interpolation.

In Vienna we can perform spatial cluster analyses (case 2), because there is a low number of temperature profile measurements available. Figure 7 shows the location of the temperature logs in respect to hydrogeological conditions of the underground up to 300 m. The hydrogeological units are summarized in 8 “homogenous areas” for simplification reasons. Now,

all linear models with similar gradients and historic surface temperatures can be clustered in respect to the homogenous areas.

In Vienna 4 clusters had been defined, allocating 5 profiles to cluster 1, 3 profiles to cluster 2, 1 profile to cluster 3 and 3 profiles to cluster 4, see Figure 7 and Table 2. For every cluster the average value of the gradient and the historic surface temperature ( $T0_{hist}$ ) are calculated and represents the **basic linear model** for the cluster. In Figure 8 the basic linear model is plotted for every cluster in comparison to the measured temperature profiles.

Because the pilot area Vienna (PA Vienna) is located in the same hydrogeological homogenous area, the basic linear model of cluster 2 can be applied for the whole pilot area, with  $T0_{hist} = 10.7\text{ }^{\circ}\text{C}$  and a gradient of  $0.02\text{ K/m}$ .



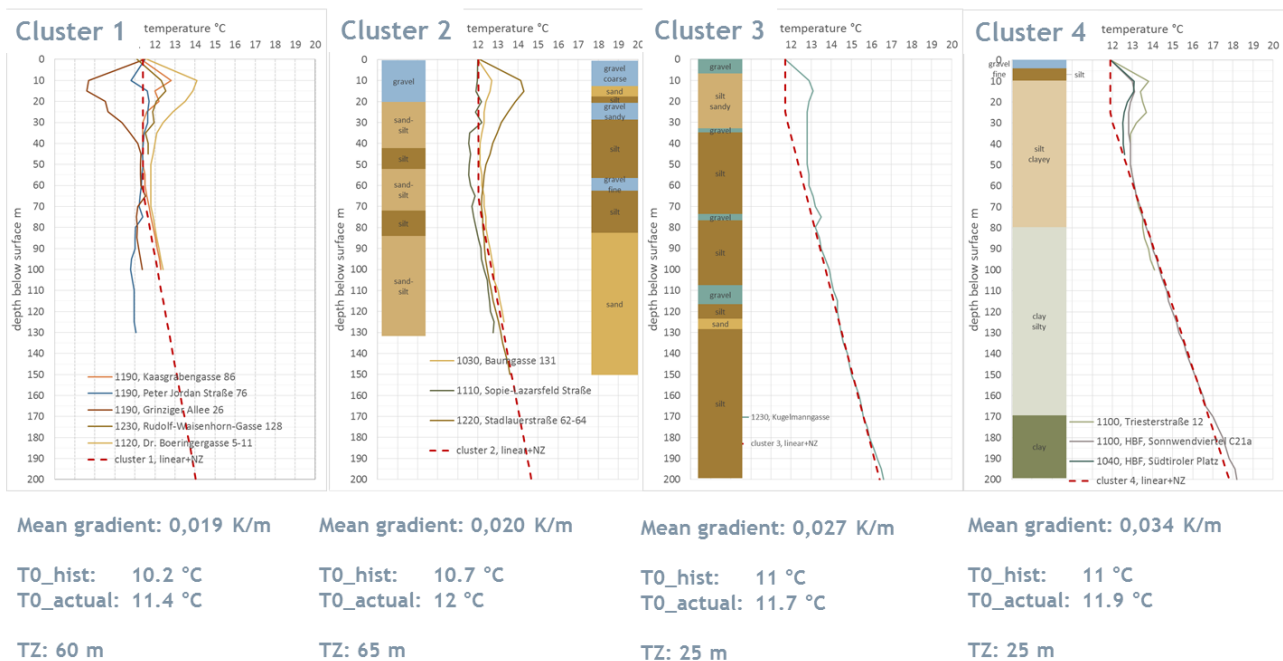
**Figure 7: Location of the temperature profiles and allocation to cluster areas for Vienna**

**Table 2: Results of step 1 and 2: basic linear models (gradient,  $T0_{hist}$ ) allocated to 4 clusters**

name	cluster	$T0_{actual}$ $^{\circ}\text{C}$	$T0_{hist}$ $^{\circ}\text{C}$	gradient_80 $^{\circ}\text{C/m}$	$R^2$	MSL m
TRT#01, 1230	1	12.1				265
TRT#02, 1190	1	11.3	10.31	0.020	0.9958	246
TRT#03, 1190	1	11.5	10.90	0.000	0.0100	236
TRT#04, 1190	1	11.5	9.95	0.014	0.8929	213
TRT#05, 1120	2	11.6	10.40	0.020	1.0000	220



TRT#06, 1030	2	12	10.80	0.020	0.9891	158
TRT#07, 1110	2	12	10.53	0.018	0.9716	172
TRT#08, 1220	2	11.1	10.48	0.020	0.9843	159
TRT#09, 1230	3	11.7	11.06	0.027	0.9934	206
TRT#10, 1100	4	11.9	11.08	0.030	0.9868	210
TRT#11, 1100	4	11.9	10.49	0.038	0.9943	200
TRT#12, 1040	4	11.9				206



**Figure 8: Temperature profiles of Vienna, divided in 4 clusters with similar gradients and surface temperature**

## 2.3. step 3: surface temperature

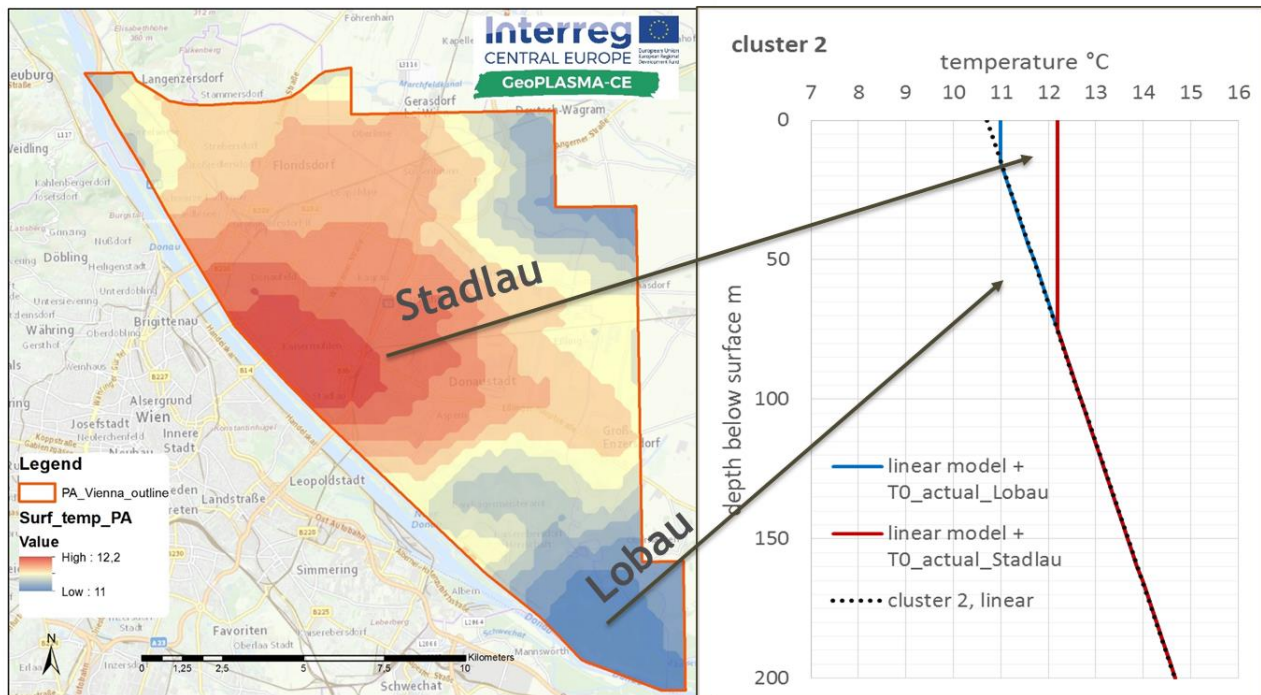
The GeoPLASMA-CE team recommends to use the mean annual surface temperature based on satellite data. The dataset can be downloaded from <http://www.geodati.fmach.it/eurolst.html>. It is derived from daily MODIS LST data, for period 2000 - 2013. The spatial resolution is 250 m, temperature sensitivity 0.1 deg C. The data are free as long as our derived products remain also free (Open Database License).

In the last workflow step we derived one **basic linear model** for the whole PA Vienna (same cluster area). In step 3 we can now update the basic linear model for every grid (raster) cell of the pilot area with the actual surface temperature. That means, that every grid cell has now an individual temperature profile starting at the individual actual surface temperature and prolong it into depth, until the basic linear model is met. Hence, also the depth of the transient zone (z2) is now fixed individually for every grid point. The formula to calculate z2 is given in the next workflow step.

The basic linear model (grad T and the historic surface temperature) will be the same for the whole cluster area. Figure 9 demonstrates this workflow step for two selected locations (grid



cells) in the PA Vienna. At the location “Stadlau” the actual surface temperature is 12.1 °C, hence the depth of the transient zone  $z_2$  is in 70.5 m. At the location “Lobau” the actual surface temperature is 11.0 °C, resulting in  $z_2 = 15.1$  metre. Below the transient zone the gradient with its offset at surface ( $T_0_{\text{historic}}$ ) is the same for every grid point.



**Figure 9: Land Surface Temperature (LST) data for PA Vienna as an example. LEFT: Low LST are in “Lobau” with 11 °C and the high LST are in “Stadlau” with 12.1 °C. RIGHT: The actual surface temperature (LST) is added to the linear model of cluster 2 for every grid point. Two profiles (Lobau, Kaisermuehlen) are plotted as example.**

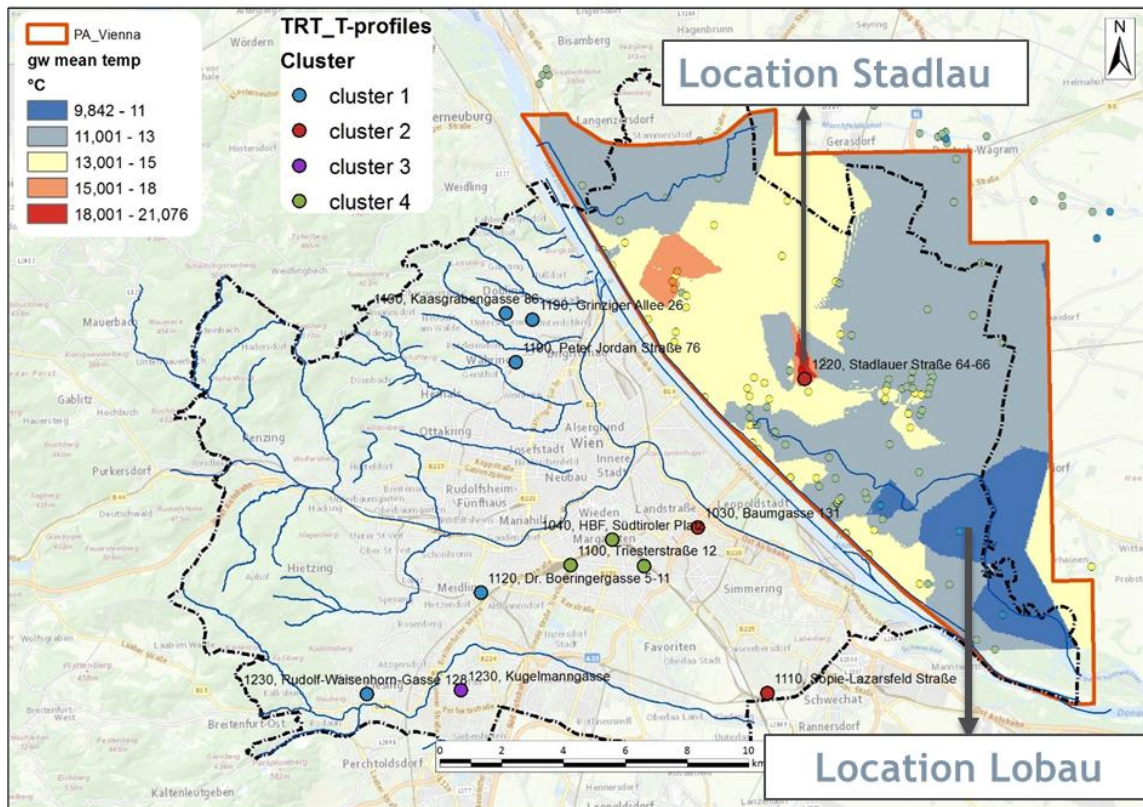
## 2.4. step 4: transient zone

This workflow step can only be done, if an additional temperature information (measurements) is available in the transient zone as a map. As example, at PA Vienna, the average annual temperature for a shallow groundwater body is available as a map, see Figure 10. For every grid cell (with additional temperature information) the groundwater temperature and the depth of the measurement has to be available. The depth of  $z_2$  at PA Vienna was set to the middle of the aquifer. Now, for every grid cell, an **enhanced model** can be defined by the five parameters:  $T_0_{\text{actual}}$ ,  $T_1$ ,  $z_1$ ,  $\text{grad } T$  and  $T_0_{\text{historic}}$ .  $T_2$  is set to  $T_0_{\text{actual}}$  by definition and  $z_2$  can be calculated for every grid point by the following formula:

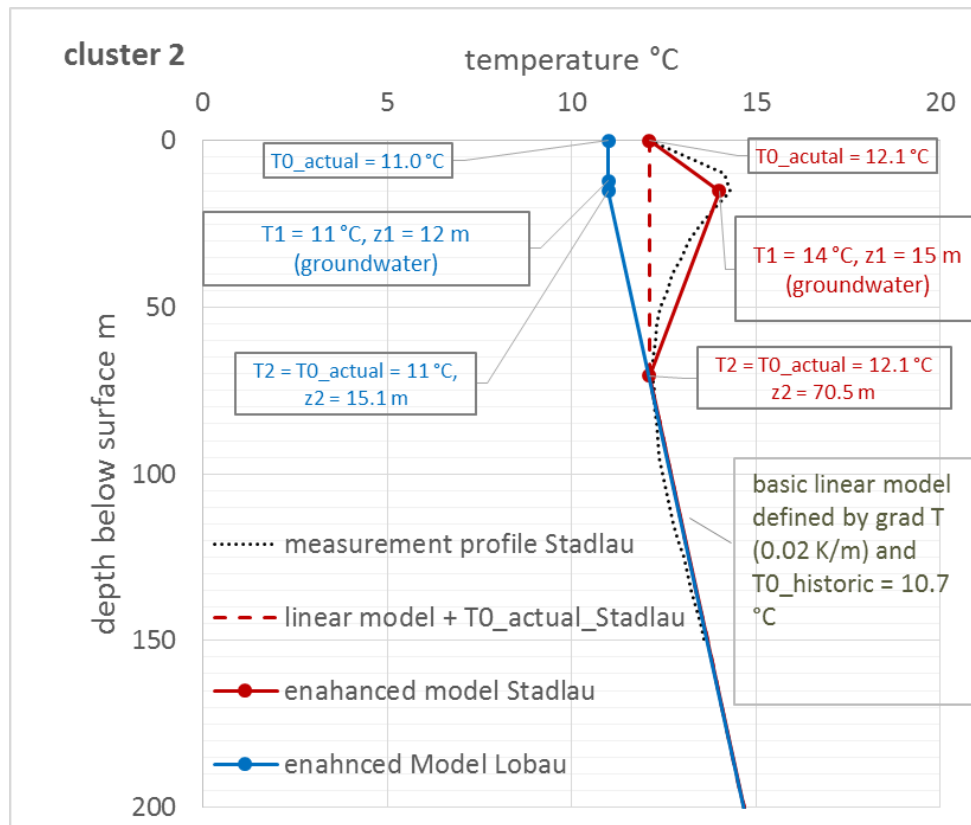
$$z_2 = \frac{T_2 - T_0_{\text{historic}}}{\text{grad } T} = \frac{T_0_{\text{actual}} - T_0_{\text{historic}}}{\text{grad } T}$$

Figure 11 demonstrates this workflow step with a plot of the two selected locations in PA Vienna. At location “Stadlau” the groundwater temperature is really warm (average annual temperature = 14 °C). The actual surface temperature is about 12.1 °C. With the basic linear model ( $\text{grad } T = 0.02$ ,  $T_0_{\text{historic}} = 11.7$  °C) the depth of  $z_2$  can be calculated with  $z_2 = (12.1 - 10.7)/0.01985 = 70.5$  m. At this location also a measured temperature profile is available and

can be compared with the enhanced model in Figure 11. At the location “Lobau” the measured annual mean groundwater temperature at  $z_2 = 12$  m is  $11^\circ\text{C}$  and equal to the actual mean surface temperature (undisturbed). The enhanced model, in this case, is equal to the linear model with actual surface temperature. The depth of  $z_2$  can be calculated with  $z_2 = (11 - 10.7)/0.01985 = 15.1$  m



**Figure 10: Groundwater temperature map for creating enhanced temperature models to every grid cell**



**Figure 11: Enhanced subsurface temperature model for two selected locations in Vienna**

## 2.5. step 5: calculate output parameters

In this workflow steps the defined GeoPLASMA\_CE output parameters has to be calculated from the transient or the enhanced model: the mean temperature and mean gradients of defined depth intervals. Figure 12 shows the models in comparison.

Two cases can be defined:

- 1) no additional temperature information in the transient zone:

The calculation of the output parameters can be done with the aid of the predefined Excel sheet in Annex 6a, by giving at least the main input data ( $T_{0\_historic}$ ,  $grad T$ ,  $T_{0\_actual}$ ). The columns for the optionally input data ( $T_1$ ,  $z_1$ ) can be left empty, the transient model is considered.  $z_2$  can be calculated, as defined in step 4.  $T_2$  is equal to  $T_{0\_actual}$ . The output parameters  $T_{mean}$  and  $gradT_{mean}$  can be calculated down to the depth  $d$  (valid for  $d > z_2$ ):

$$T_{mean}(d) = \frac{1}{d} \cdot \left[ T_0 \cdot d + \frac{T(d) + T_2}{2} \cdot (d - z_2) \right] = \frac{1}{d} \cdot \left[ T_0 \cdot d + \frac{T_2 + T_2 + grad T \cdot (d - z_2)}{2} \cdot (d - z_2) \right]$$

$$= \frac{1}{d} \cdot \left[ T_0 \cdot d + T_2 \cdot (d - z_2) + grad T \cdot \frac{(d - z_2)^2}{2} \right]$$

$$gradT_{mean}(d) = \frac{1}{d} \cdot [grad T \cdot (d - z_2)]$$

2) with temperature information in the transient zone ( $T_1, z_1$ ):

When the columns for the optionally input data ( $T_1, z_1$ ) are filled, the enhanced model is considered automatically and the following formula is applied. It gives the weighted average of the subsurface temperature and the temperature gradient for depths ( $d$ ) greater than  $z_2$ :

$$T_{mean}(d) = \frac{1}{d} \cdot \left[ \frac{T_0 + T_1}{2} \cdot z_1 + \frac{T_1 + T_2}{2} \cdot (z_2 - z_1) + T_2 \cdot (d - z_2) + grad\ T \cdot \frac{(d - z_2)^2}{2} \right]$$

$$gradT_{mean}(d) = \frac{1}{d} \cdot \left[ gradT_{0,1} \cdot z_1 + gradT_{1,2} \cdot (z_2 - z_1) + grad\ T \cdot (d - z_2) \right]$$

Note that in both cases  $T_2$  can be set to the same value as the surface temperature  $T_0$ .

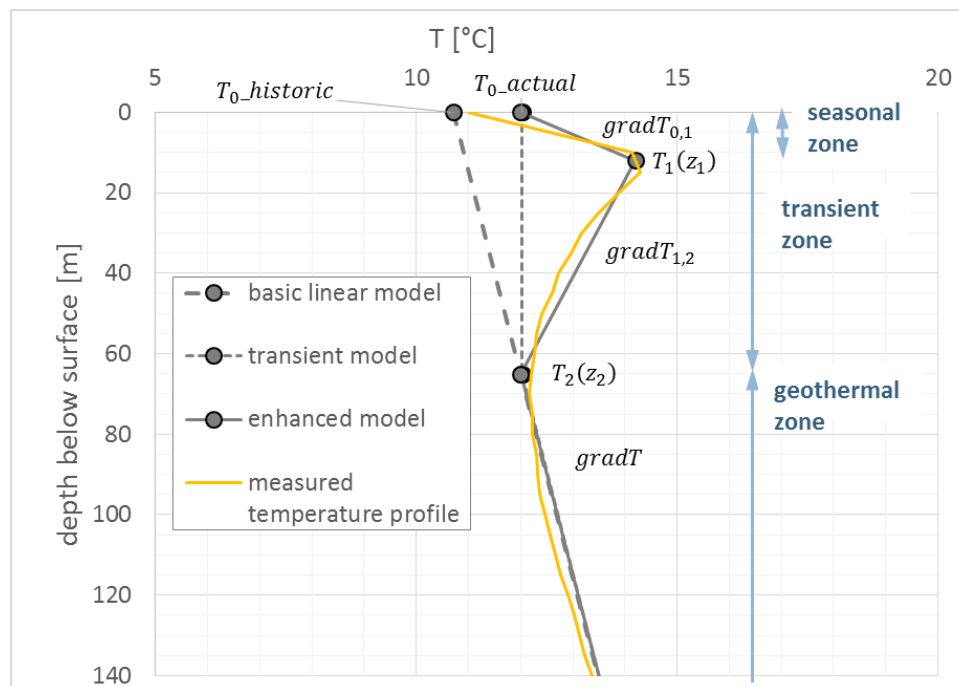
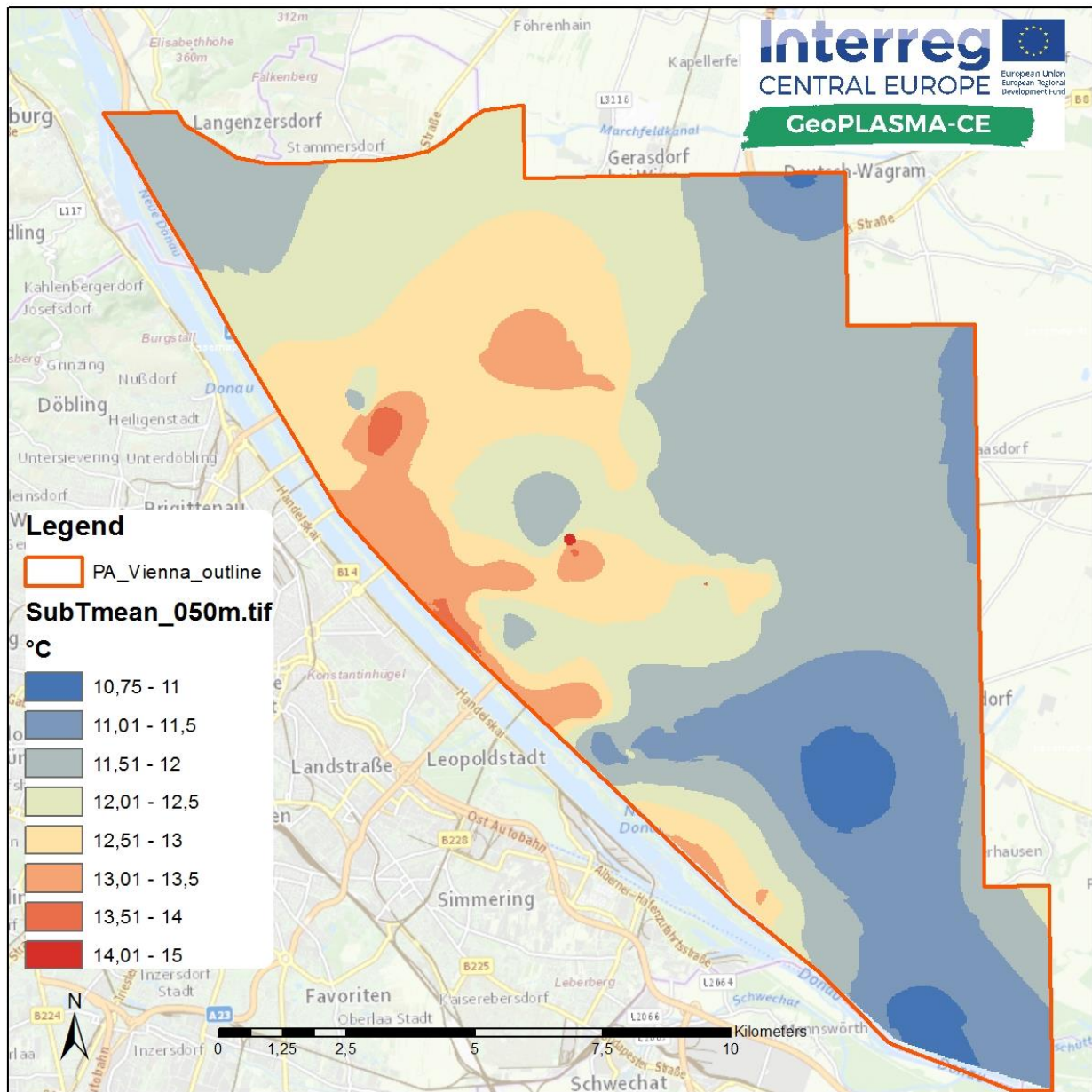


Figure 12: Linear approximation of a transient temperature signal (yellow line) in three steps: “basic linear model” from geothermal gradient extrapolation with  $T_{0\_historic}$ , “transient model” with actual surface temperature  $T_{0\_actual}$  and “enhanced model” with additional temperature information within the transient zone. The depth range of a transient signal caused by temperature changes on the surface over decades (climate change, urban heat island) is dependent primary on the signal duration. e.g. transient signal for 50 years can reaches depths of up to 100 meters below surface.



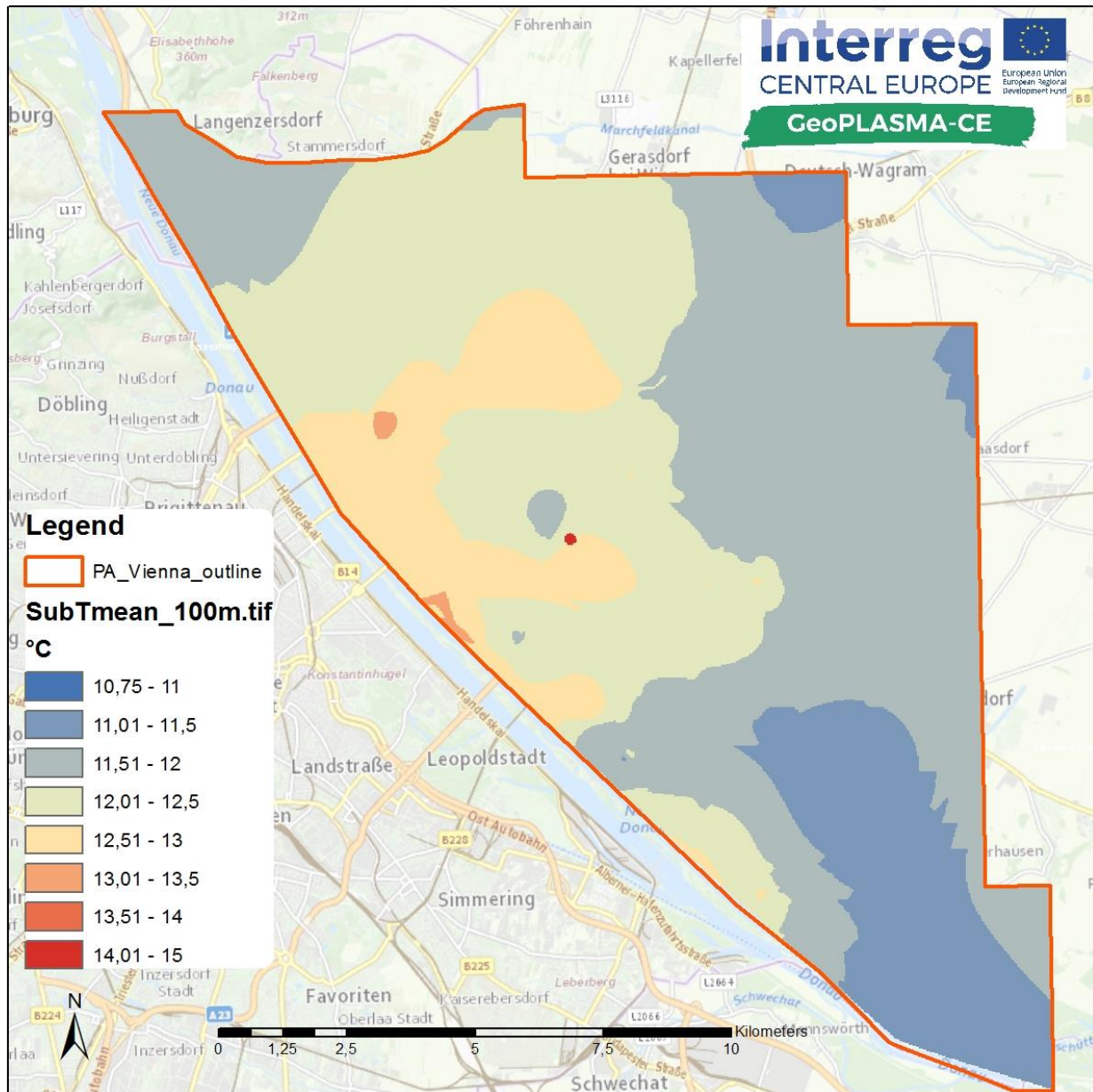
## 2.6. Results (Output maps) for PA Vienna

Figure 13, Figure 14, Figure 15 and Figure 16 shows the final GeoPLASMA-CE output maps for the pilot area Vienna (PA Vienna) as an example.

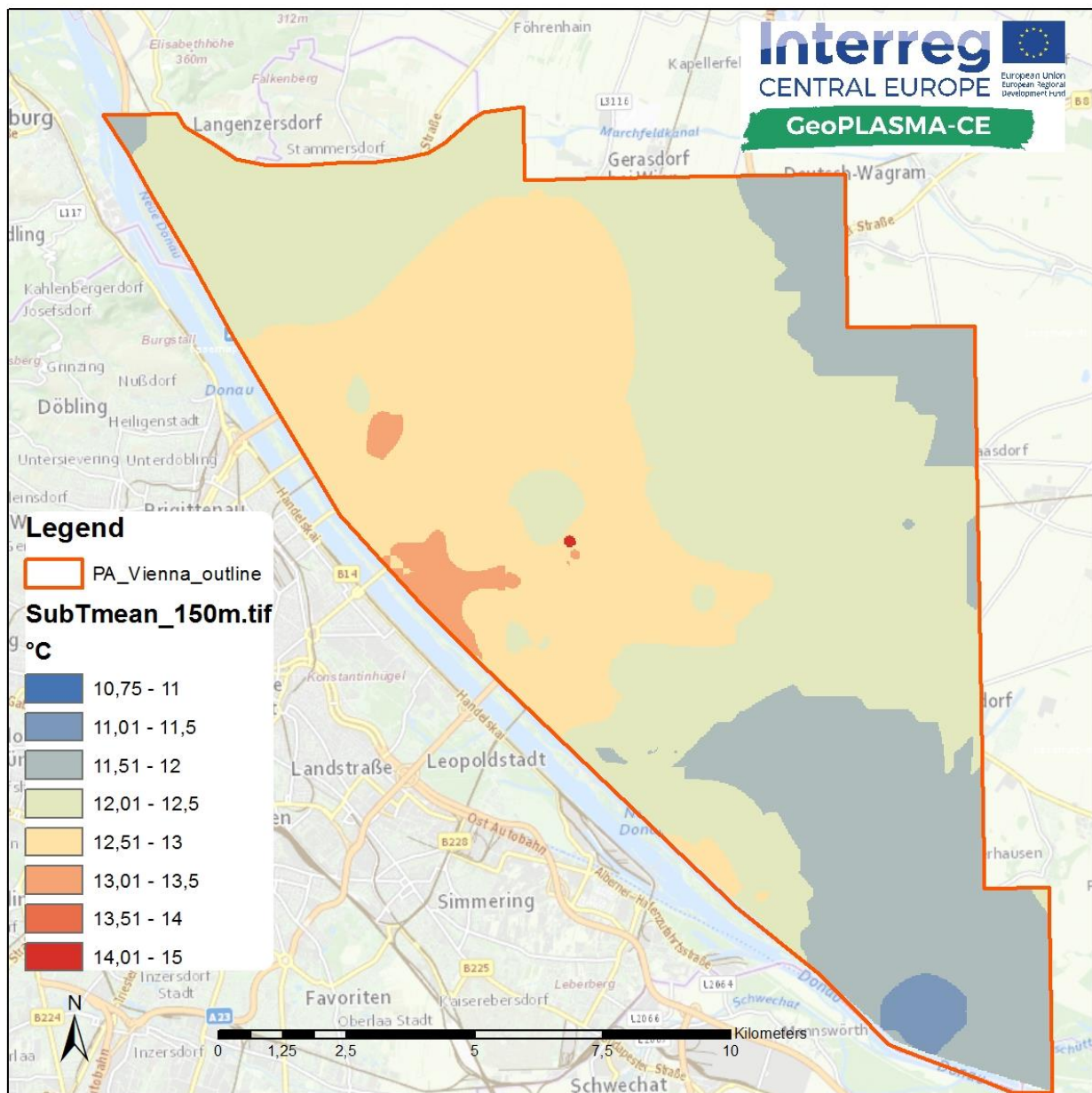


**Figure 13: Subsurface temperature map as an GeoPLSAMA-CE output for PA Vienna, showing average temperature from 0-50m below surface**

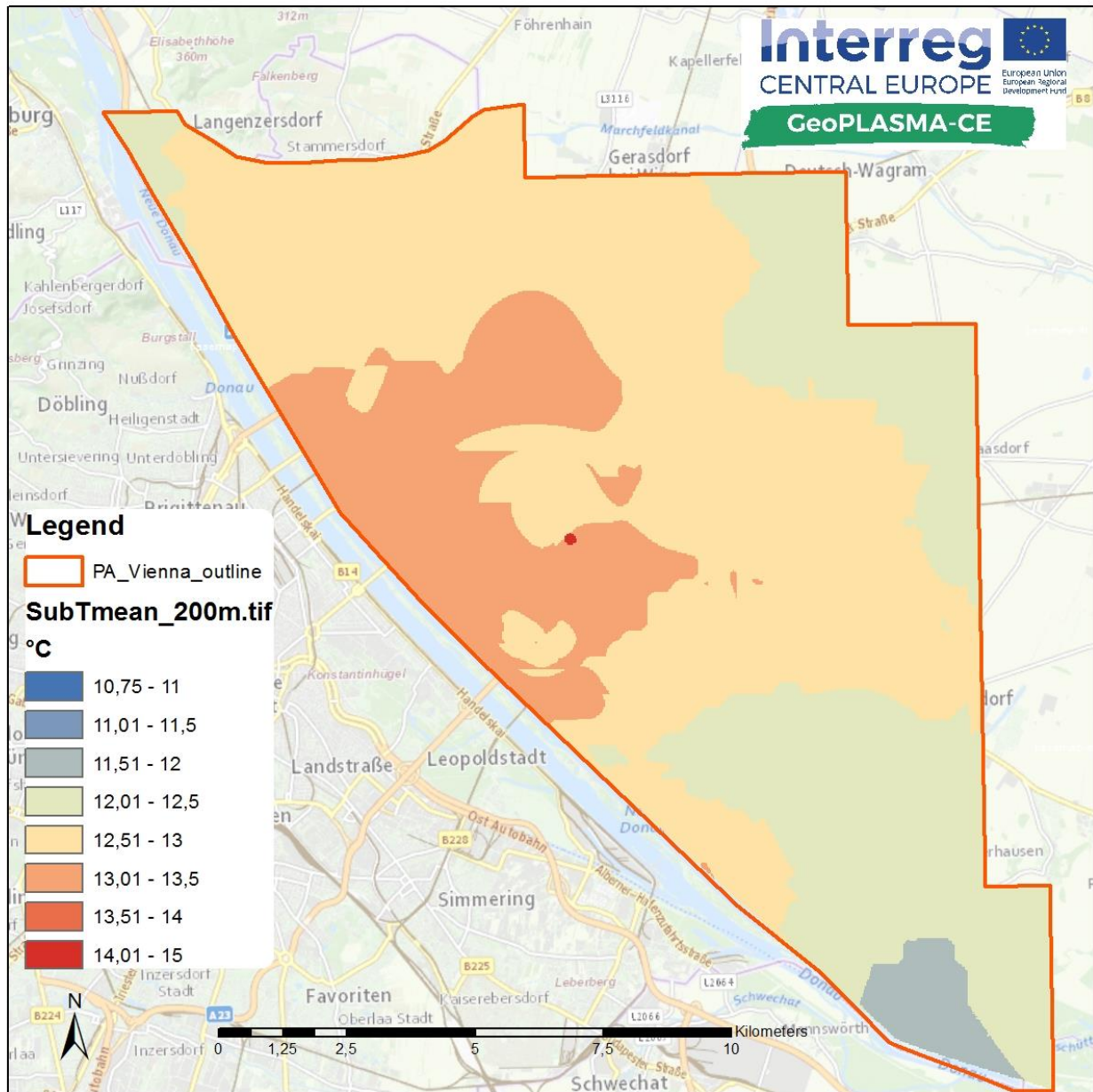




**Figure 14: Subsurface temperature map as an GeoPLSAMA-CE output for PA Vienna, showing average temperature from 0-100m below surface**



**Figure 15: Subsurface temperature map as an GeoPLSAMA-CE output for PA Vienna, showing average temperature from 0-150m below surface**



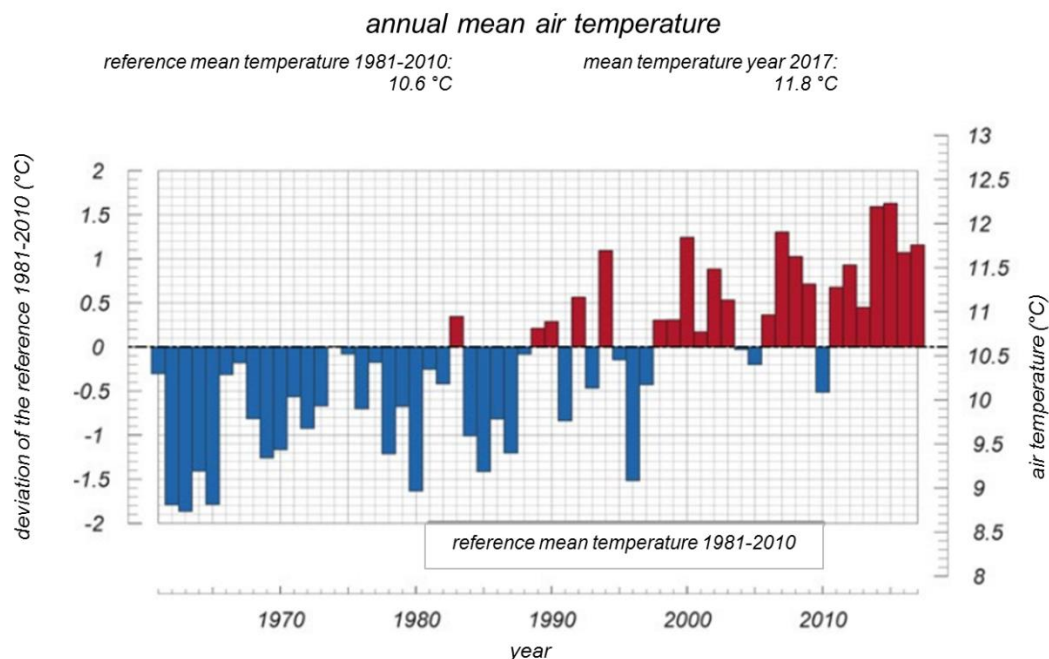
**Figure 16: Subsurface temperature map as an GeoPLSAMA-CE output for PA Vienna, showing average temperature from 0-200m below surface**



### 3. Additional information and background

#### 3.1. Climate change effect for the last 60 years at the city of Vienna

Figure 17 shows the mean annual air temperature development of the last years for the municipal Vienna. The air temperature increased in the last 60 years in the range of 2 Kelvin. It can be assumed, that this rise can be applied to the surface soil temperature equally. The absolute mean values of the air temperature is about 1 Kelvin lower than the soil temperature at surface (air-earth coupling effect), depending on the land cover and sun exposure.



**Figure 17: climate change effect of the air temperature in Vienna, source: Ropac et al, 2018, translated to English**

#### 3.2. Gaussian model: calculate the effect of a sudden surface temperature change on the surface to the underground

Long range temperature changes of the average surface temperature of the last century due to urbanization “urban heat island” or “climate change”, effects the subsurface temperature of the shallow underground (up to 200 m depth). Especially in urban areas, this effect can play an important role to the shallow temperature regime.

The effect of a transient temperature signal to the subsurface can be estimated with an analytic solution of the partial differential heat transfer equation of a sudden temperature rise (step function) on the surface of a semi-infinite plane wall of homogenous or layered material.

The solution can be simply described by defining two auxiliary variables “ $\Theta$ ” and “ $\eta$ ” and with the help of the inverse Gaussian error function “erfc”, as follows:



$$\Theta = \frac{T(z, \tau) - T_0}{T_j - T_0} = \operatorname{erfc}(\eta) = \operatorname{erfc}\left(\frac{z}{2\sqrt{a \cdot \tau}}\right)$$

Hence, the higher the distance from the surface (*depth*  $z$ ) or the lower the impact time  $\tau$  of the temperature rise ( $T_j - T_0$ ), the higher is  $\eta$  and the smaller is the temperature disturbance at depth  $z$  [ $T(z, \tau) - T_0$ ]. Of course, the impact of the temperature disturbance is also dependent on the thermal diffusivity ( $a$ ) of the underground, while  $T_0$  is the initial temperature of the surface before the temperature step. More details of the deviations can be found e.g. Herwig et al, 2006

In simple words: The formula gives the temperature change to the subsurface, if there has been a sudden temperature step on the surface since a given time ( $\tau$ ).

## 4. References

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